

Reevaluation of Higgs-Mediated μ - e Transition in the MSSMJunji Hisano^{a,b,c}, Shohei Sugiyama^{a,c}, Masato Yamanaka^{a,d},
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Kamigamo, Kita-Ku, Kyoto, 603-8555, Japan***Abstract**

In this letter, we reevaluated the Higgs-mediated contribution to $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, and μ - e conversion in nuclei in the MSSM, assuming left-handed sleptons have flavor-mixing mass terms. Contrary to previous works, it is found that Barr-Zee diagrams including top quark give dominant contribution to $\mu \rightarrow e\gamma$, and those including bottom quark and tau lepton are also non-negligible only when $\tan\beta$ is large. As a result, the Higgs-mediated contribution dominates over the gaugino-mediated contribution at one-loop level in $\mu \rightarrow e\gamma$ when $M_{\text{SUSY}}/m_{A^0} \gtrsim 50$, irrespectively of $\tan\beta$ as far as $\tan\beta$ is not large. Here, M_{SUSY} and m_{A^0} are a typical mass scale of the SUSY particles and the CP-odd Higgs boson mass, respectively. Ratio of branching ratios for $\mu \rightarrow e\gamma$ and μ - e conversion in nuclei is also evaluated by including both the gaugino- and Higgs-mediated contributions to the processes. It is found that the ratio is sensitive to $\tan\beta$ and M_{SUSY}/m_{A^0} when $M_{\text{SUSY}}/m_{A^0} \sim (10 - 50)$ and $\tan\beta \gtrsim 10$.

1 Introduction

Charged lepton-flavor violating (cLFV) processes, such as $\mu \rightarrow e\gamma$, are sensitive to physics beyond the standard model (SM) [1]. While the lepton-flavor conservation is not exact in nature due to finite neutrino masses, cLFV processes are quite suppressed in the standard model. Thus, searches for cLFV processes are a unique window to physics beyond the SM, especially, at TeV scale.

Now the MEG experiment is searching for $\mu \rightarrow e\gamma$ [2], it would reach to $\sim 10^{-13}$ for the branching ratio on the first stage, which is improvement of two orders of magnitude compared with the current bound. The COMET and Mu2e experiments [3, 4], which are searches for μ - e conversion in nuclei, are being planed in J-PARC and Fermilab, respectively. It is argued that they would reach to $\sim 10^{-16}$ for branching ratio of μ - e conversion with target Al. Here, branching ratio of μ - e conversion is ratio of μ - e conversion rate over muon capture rate. Searches for $\mu \rightarrow e\gamma$ and μ - e conversion in nuclei are complementary to each other in studies of physics beyond the SM since those processes may be induced by different types of processes.

The minimal supersymmetric (SUSY) standard model (MSSM) is a leading candidate for physics beyond the SM, and cLFV processes are extensively studied in the model. SUSY-breaking slepton mass terms are lepton-flavor violating. It is noticeable that ratios of branching ratios for cLFV processes would give information of mass spectrum in the MSSM, since dominant diagrams in cLFV processes depend on the mass spectrum.

When SUSY particle masses are $\lesssim O(1)$ TeV, the muon LFV processes, such as μ - e transition processes, $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and μ - e conversion in nuclei, are generated by the gaugino-mediated contribution, which is generated by one-loop diagrams of gauginos and sleptons (and Higgsinos). Branching ratios for the cLFV processes due to the gaugino-mediated contribution are suppressed by $1/M_{\text{SUSY}}^4$, since the effective dipole interaction is dominant in the cLFV processes. Here, M_{SUSY} is a typical mass scale of the SUSY particles. On the other hand, when $M_{\text{SUSY}} \gtrsim O(1)$ TeV, the Higgs-mediated contribution to the processes could be sizable. The non-holomorphic LFV correction is generated to Yukawa coupling of the Higgs bosons at one-loop level, and it is not suppressed by M_{SUSY} [6]. Branching ratio of μ - e conversion in nuclei is more sensitive to the Higgs-mediated contribution [7]. Thus, ratio of branching ratios for $\mu \rightarrow e\gamma$ and μ - e conversion in nuclei is a good observable to constrain mass spectrum in the MSSM, since it is sensitive to whether the gaugino-mediated or Higgs-mediated contribution is dominant.

In this letter we systematically calculate the Higgs-mediated contributions to cLFV reactions in the MSSM, and clarify the dominant process in each cLFV reaction. For this purpose, we first evaluate the Higgs-mediated contribution to $\mu \rightarrow e\gamma$ in the MSSM. Barr-Zee diagrams give dominant contribution to $\mu \rightarrow e\gamma$ among various diagrams though those are of higher order. We systematically evaluate those diagrams, and find that Barr-Zee diagrams including top quark give the largest contribution, and the branching ratio for $\mu \rightarrow e\gamma$ induced by the Barr-Zee diagrams is approximately proportional to $\tan^2 \beta$. The angle β is defined by $\tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$. The Higgs-mediated contribution dominates over the gaugino-mediated one in $\mu \rightarrow e\gamma$ when $M_{\text{SUSY}}/m_{A^0} \gtrsim 50$, which is

almost insensitive to $\tan\beta$. Here, m_{A^0} is the CP-odd Higgs boson mass in the MSSM.

Using this result, we evaluate ratio of branching ratios for $\mu \rightarrow e\gamma$ and μ - e conversion in nuclei. When the Higgs-mediated contribution is dominant, the ratio of the branching ratios is scaled by $\tan^4\beta$. It is found that the ratio is quite sensitive to $\tan\beta$ and M_{SUSY}/m_{A^0} when $M_{\text{SUSY}}/m_{A^0} \sim (10 - 50)$ and $\tan\beta \gtrsim 10$. We also check that the ratio of the branching ratios for $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ is insensitive to them.

In Refs. [8, 9] the Higgs-mediated contribution to the μ - e transition processes in the MSSM is discussed. It is argued that when the Higgs-mediated contribution is dominant, the Barr-Zee diagram including W boson is dominant and branching ratio of $\mu \rightarrow e\gamma$ is scaled by $\tan^4\beta$, not $\tan^2\beta$. This is obviously overestimated. We clarify what is wrong in their deviation.

We assume that left-handed sleptons have flavor-mixing mass terms in this letter, simply because this setup is well-motivated from the SUSY seesaw model [5]. Extension to more general cases will be given elsewhere.

This letter is organized as follows. In the next section we evaluate the Higgs-mediated contribution for $\mu \rightarrow e\gamma$. We show ratio of the Higgs-mediated and gaugino-mediated contributions. In Sec. 3, we discuss the Higgs-mediated contributions to $\mu \rightarrow 3e$ and μ - e conversion in nuclei, and evaluate ratios among the cLFV processes. Sec. 4 is devoted to conclusions and discussion.

2 Higgs-mediated contribution to $\mu \rightarrow e\gamma$ in the MSSM

In the MSSM, LFV in the Higgs coupling originates from the non-holomorphic correction to Yukawa interaction of charged leptons [6]. By including the correction due to one-loop diagrams of gaugino and sleptons, the effective Yukawa coupling is given as follows:

$$-\mathcal{L}_{\text{eff}} = \bar{e}'_{Ri} y_{ei} H_1^0 e'_{Li} + \bar{e}'_{Ri} y_{ei} \left(\epsilon_1^{(i)} \delta_{ij} + \epsilon_2^{(ij)} \right) H_2^{0*} e'_{Lj} + \text{h.c.}, \quad (1)$$

where y_{ei} stands for the i -th charged-lepton Yukawa coupling constant at tree level and e'_{Ri} and e'_{Li} represent right-handed and left-handed leptons, respectively, in a basis where the tree-level lepton Yukawa matrix is diagonal. The non-holomorphic interaction $\epsilon_2^{(ij)}$ ($i \neq j$) is generated by flavor-violating slepton mass terms. As mentioned in introduction, we assumed that left-handed sleptons have flavor-violating mass terms. We parametrize $\epsilon_2^{(ij)}$ with mass insertions (MIs) parameters, $\delta_{ij}^{LL} = (\Delta m_{l_L}^2)_{ij} / \tilde{m}_{l_L}^2$, where $(\Delta m_{l_L}^2)_{ij}$ is off-diagonal element of left-handed slepton mass matrix and \tilde{m}_{l_L} is an average left-handed slepton mass. When the SUSY-breaking mass parameters in the MSSM are taken to be a common value (M_{SUSY}), the non-holomorphic corrections $\epsilon_1^{(i)}$ and $\epsilon_2^{(ij)}$ are reduced to

$$\begin{aligned} \epsilon_1^{(i)} &= \frac{g_Y^2}{64\pi^2} - \frac{3g_2^2}{64\pi^2}, \\ \epsilon_2^{(ij)} &= \left(-\frac{g_Y^2}{64\pi^2} + \frac{g_2^2}{64\pi^2} \right) \delta_{ij}^{LL}. \end{aligned} \quad (2)$$

Note that $\epsilon_1^{(i)}$ and $\epsilon_2^{(ij)}$ do not vanish even in a limit of large masses of SUSY particles. This is quite different from LFV effective dipole operators induced by the gaugino-slepton loops, whose coefficients are suppressed by masses of internal SUSY particles.

In a mass-eigenstate basis for both leptons and Higgs bosons, \mathcal{L}_{eff} for μ - e transition is described as [6]

$$-\mathcal{L}_{\text{eff}}^{\mu-e} = \frac{m_\mu \Delta_{\mu e}^L}{v \cos^2 \beta} (\bar{\mu} P_L e) [\cos(\alpha - \beta) h^0 + \sin(\alpha - \beta) H^0 - i A^0] + \text{h.c.} , \quad (3)$$

where h^0 and H^0 are the CP-even Higgs fields ($m_{h^0} < m_{H^0}$), and A^0 is the CP-odd Higgs field. The LFV parameter $\Delta_{\mu e}^L$ is given by $\Delta_{\mu e}^L = \epsilon_2^{(\mu e)} / (1 + \epsilon_1^{(\mu)} \tan \beta)^2$. When we treat $\epsilon_1^{(\mu)}$ and $\epsilon_2^{(\mu e)}$ as a perturbation, we may neglect $\epsilon_1^{(\mu)}$ of the denominator at the first order. In this letter, we set $\Delta_{\mu e}^L = \epsilon_2^{(\mu e)}$.

In the MSSM, LFV interaction of h^0 in Eq. (3) vanishes when the masses of H^0 and A^0 go to infinity, since $\cos(\alpha - \beta)$ behaves as

$$\cos(\alpha - \beta) \sim \frac{-2m_{Z^0}^2}{m_{A^0}^2 \tan \beta}. \quad (4)$$

This comes from a fact that SM does not have LFV and the light Higgs boson h^0 becomes SM-like in above limit. Therefore the contributions from H^0 and A^0 should be included in cLFV processes.

Now we consider the Higgs-mediated contribution to $\mu \rightarrow e \gamma$ in the MSSM. Effective amplitude for $\mu \rightarrow e \gamma$ is parametrized as

$$T = e \epsilon^{*\mu}(q) \bar{u}_e(p - q) \left[m_\mu i \sigma_{\mu\nu} q^\nu (A^L P_L + A^R P_R) \right] u_\mu(p) , \quad (5)$$

and branching ratio of $\mu \rightarrow e \gamma$ is derived as $\text{BR}(\mu \rightarrow e \gamma) = (48\pi^3 \alpha_{\text{em}} / G_F^2) (|A^L|^2 + |A^R|^2)$. Here, $\alpha_{\text{em}} (\equiv e^2 / 4\pi)$ is the fine structure constant and G_F is the Fermi constant. While this amplitude could be induced at one-loop level (Fig. 1), it is suppressed by three chiral flips, *i.e.*, one chirality flip in the lepton propagator and two lepton Yukawa couplings. Indeed two-loop diagrams may be significant contribution. As shown in Fig. 2, two-loop diagrams, called as Barr-Zee diagrams, involve only one chiral flip (from lepton Yukawa coupling), and hence their contribution is much larger than that at one-loop level.

Following Refs. [8, 9], we consider Barr-Zee diagrams which involve effective γ - γ - ϕ^0 vertices ($\phi^0 = h^0, H^0$, and A^0). The effective vertices are induced by heavy fermion or weak gauge/Higgs boson loops. Barr-Zee diagrams involving bottom- and top-quark

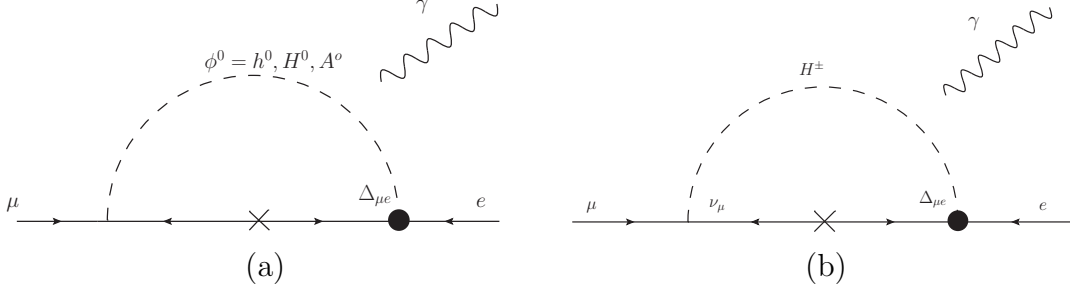


Figure 1: $\mu \rightarrow e\gamma$ induced by Higgs boson exchange at one loop level.

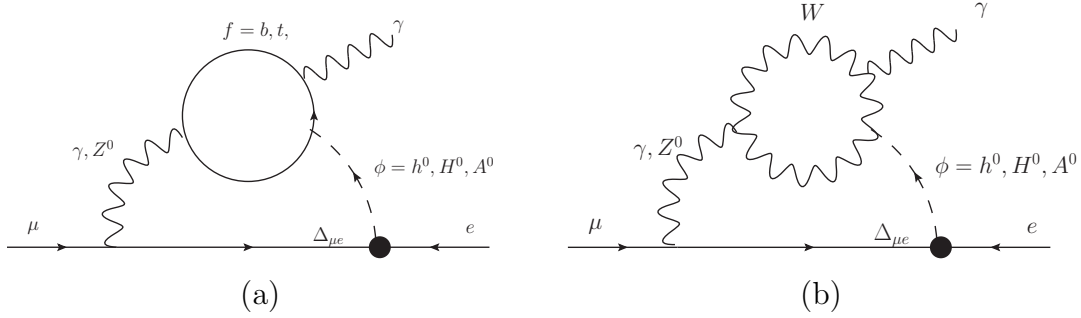


Figure 2: Examples of two-loop Barr-Zee diagrams induced by Higgs exchange.

loops (Fig. 2 (a)) give contributions to the coefficient A^R in Eq. (5) as

$$\begin{aligned}
A_{\text{BZ}(b)}^R &= \frac{2\sqrt{2}G_F\alpha_{\text{em}}N_cQ_b^2}{16\pi^3}\Delta_{\mu e}^L \\
&\times \left[-\frac{\cos(\alpha-\beta)\sin\alpha}{\cos^3\beta}f(z_{h^0}^b) + \frac{\sin(\alpha-\beta)\cos\alpha}{\cos^3\beta}f(z_{H^0}^b) + \frac{\sin\beta}{\cos^3\beta}g(z_{A^0}^b) \right], \\
A_{\text{BZ}(t)}^R &= \frac{2\sqrt{2}G_F\alpha_{\text{em}}N_cQ_t^2}{16\pi^3}\Delta_{\mu e}^L \\
&\times \left[\frac{\cos(\alpha-\beta)\cos\alpha}{\cos^2\beta\sin\beta}f(z_{h^0}^t) + \frac{\sin(\alpha-\beta)\sin\alpha}{\cos^2\beta\sin\beta}f(z_{H^0}^t) + \frac{1}{\sin\beta\cos\beta}g(z_{A^0}^t) \right].
\end{aligned} \tag{6}$$

Here, N_c is color factor, and $Q_{b(t)}$ represents electric charge for bottom (top) quark. $z_{\phi^0}^q = m_q^2/m_{\phi^0}^2$ for $\phi^0 = h^0, H^0, A^0$ and $q = b, t$. Similarly, we calculate the coefficient for tau-lepton loop by substituting $N_c = 1$, and replacing Q_b, m_b to Q_τ, m_τ . The functions $f(z)$ and $g(z)$ are called Barr-Zee integrals, whose explicit forms and asymptotic behaviors are given in Appendix. For $m_{A^0} \gg m_{Z^0}$ and $\tan\beta \gg 1$, the Barr-Zee diagram contribution

is approximated as

$$A_{\text{BZ}(t,b,\tau)}^R \simeq \frac{\sqrt{2}G_F N_c \alpha_{\text{em}}}{8\pi^3} \Delta_{\mu e}^L \times \left[\frac{Q_t^2 m_t^2}{m_{A^0}^2} \tan \beta \left(\log \frac{m_t^2}{m_{A^0}^2} \right)^2 - \frac{Q_b^2 m_b^2}{m_{A^0}^2} \tan^3 \beta \left(\log \frac{m_b^2}{m_{A^0}^2} + 2 \right) - (b \rightarrow \tau) \right]. \quad (7)$$

It is found that $\tan \beta$ and/or large logarithmic factors enhance heavy-Higgs (H^0 , A^0) contributions, and the light-Higgs (h^0) contribution is subdominant.

Similarly, the Barr–Zee contributions from loops of W^- boson (Fig. 2 (b)), Nambu–Goldstone (NG) boson G^- , and charged Higgs boson H^- are calculated. Each contribution to A^R is derived as follows,

$$A_{\text{BZ}(W^-)}^R = \frac{\alpha_{\text{em}} \sqrt{2} G_F}{16\pi^3} \frac{\sin(\alpha - \beta) \cos(\alpha - \beta)}{\cos^2 \beta} \left[F(z_{h^0}^{W^-}) - F(z_{H^0}^{W^-}) \right] \Delta_{\mu e}^L, \quad (8)$$

$$A_{\text{BZ}(G^-)}^R = \frac{-\alpha_{\text{em}} \sqrt{2} G_F}{16\pi^3} \frac{\sin(\alpha - \beta) \cos(\alpha - \beta)}{\cos^2 \beta} \left[F'(z_{h^0}^{W^-}) - F'(z_{H^0}^{W^-}) \right] \Delta_{\mu e}^L, \quad (9)$$

$$A_{\text{BZ}(H^-)}^R = \frac{\alpha_{\text{em}} \sqrt{2} G_F}{16\pi^3} \frac{1}{\cos^2 \beta} \left[\frac{\cos(\alpha - \beta)}{m_{h^0}^2} f_{h^0} F'(z_{h^0}^{H^-}) + \frac{\sin(\alpha - \beta)}{m_{H^0}^2} f_{H^0} F'(z_{H^0}^{H^-}) \right] \Delta_{\mu e}^L, \quad (10)$$

where $z_{\phi^0}^{\phi^-} = m_{\phi^-}^2 / m_{\phi^0}^2$ ($\phi^- = W^-, H^-$ and $\phi^0 = h^0, H^0$), and f_{ϕ^0} ($\phi^0 = h^0, H^0$) comes from coupling of $H^+ H^- \phi^0$,

$$\begin{aligned} f_{h^0} &= -2m_{W^-}^2 \sin(\alpha - \beta) + m_{Z^0}^2 \sin(\alpha + \beta) \cos 2\beta, \\ f_{H^0} &= 2m_{W^-}^2 \cos(\alpha - \beta) - m_{Z^0}^2 \cos(\alpha + \beta) \cos 2\beta. \end{aligned} \quad (11)$$

The functions $F(z)$ and $F'(z)$ are $3f(z) + 5g(z) + 3/4g(z) + 3/4h(z)$ and $(g(z) - f(z))/(2z)$, respectively. $h(z)$ is also given in Appendix. The CP-odd Higgs boson A^0 does not appear here if CP is conserved. The above contributions are approximated in a limit of $m_{A^0} \gg m_{Z^0}$ and $\tan \beta \gg 1$ as

$$A_{\text{BZ}(W^-)}^R \simeq \frac{\alpha_{\text{em}} \sqrt{2} G_F}{16\pi^3} \frac{2m_{Z^0}^2}{m_{A^0}^2} \tan \beta \Delta_{\mu e}^L \times \left[F(\cos^2 \theta_W) - \frac{35}{8} \frac{m_{W^-}^2}{m_{A^0}^2} \left(\log \frac{m_{W^-}^2}{m_{A^0}^2} \right)^2 \right], \quad (12)$$

$$A_{\text{BZ}(G^-)}^R \simeq \frac{-\alpha_{\text{em}} \sqrt{2} G_F}{16\pi^3} \frac{2m_{Z^0}^2}{m_{A^0}^2} \tan \beta \Delta_{\mu e}^L \times \left[F'(\cos^2 \theta_W) + \frac{1}{2} \left(\log \frac{m_{W^-}^2}{m_{A^0}^2} + 2 \right) \right], \quad (13)$$

$$A_{\text{BZ}(H^-)}^R \simeq \frac{\alpha_{\text{em}} \sqrt{2} G_F}{16\pi^3} \frac{m_{Z^0}^2}{m_{A^0}^2} \tan \beta \Delta_{\mu e}^L \times \left[(4 \cos^2 \theta_W \frac{m_{Z^0}^2}{m_{A^0}^2} - 2) F'(1) - (2 \cos^2 \theta_W - 1) \frac{m_{Z^0}^2}{m_{A^0}^2} \left(\frac{1}{6} \log \frac{m_{A^0}^2}{m_{Z^0}^2} + \frac{5}{18} \right) \right]. \quad (14)$$

Here, $F(\cos^2 \theta_W) = 7.96$, $F'(\cos^2 \theta_W) = 0.121$, and $F'(1) = 0.172$.

Notice that the diagram of H^0 including W^- loop gives contribution to A^R , which is proportional to $\tan \beta$ in a limit of $m_{A^0} \gg m_{Z^0}$ and $\tan \beta \gg 1$. This is because the $H^0 W^+ W^-$ coupling is suppressed by $\cos(\alpha - \beta) \sim -2m_{Z^0}^2/(m_{A^0}^2 \tan \beta)$ while the LFV Yukawa coupling in Eq. (3) is proportional to $\tan^2 \beta$. All contributions from Barr–Zee diagrams to A_R are proportional to $\tan \beta$ in a limit of $m_{A^0} \gg m_{Z^0}$ and $\tan \beta \gg 1$. The exception is those including the bottom-quark and tau-lepton loops, which tend to be subdominant in moderate $\tan \beta$. It is argued in Refs. [8, 9] that the diagram of H^0 including W^- loop gives a contribution proportional to $\tan^2 \beta$ and that it is the largest among the Barr–Zee diagrams. However, using corrected $\tan \beta$ dependence, this W^- loop contribution is no longer the largest one, as will be shown below.

We also include Barr–Zee diagrams including H^- loop, though the contribution is also subdominant. It is also proportional to $\tan \beta$ at most, and coefficient for its logarithmic term is suppressed by $m_{Z^0}^2/m_{A^0}^2$.

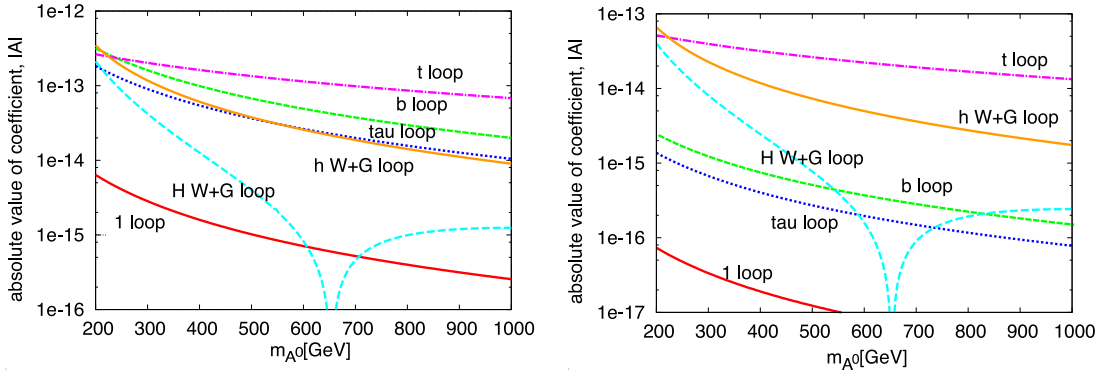


Figure 3: Absolute values of coefficients for Higgs-induced dipole operator, $|A^R|$, as a function of CP-odd Higgs boson mass m_{A^0} . We show those from diagrams including heavy-fermion loops, diagrams of light- and heavy-Higgs bosons including W^- and G^- loops. For comparison, one-loop contribution to A^R is also shown. Here, left figure is for $\tan \beta = 50$ and right one is for $\tan \beta = 10$. We took $\Delta_{\mu e}^L = 5 \times 10^{-6}$, $m_{h^0} = 120$ GeV, $m_t(m_{Z^0}) = 181$ GeV, and $m_b(m_{Z^0}) = 3.0$ GeV.

Fig. 3 shows each contribution to A^R as a function of CP-odd Higgs boson mass m_{A^0} for $\tan \beta = 50$ (left) and $\tan \beta = 10$ (right). Here, $\Delta_{\mu e}^L = 5 \times 10^{-6}$, $m_{h^0} = 120$ GeV, $m_t(m_{Z^0}) = 181$ GeV, and $m_b(m_{Z^0}) = 3.0$ GeV. Barr–Zee diagrams including top-quark loop give dominant contribution to $\mu \rightarrow e\gamma$, and the bottom-quark one is also sizable only when $\tan \beta$ is large. The W^- and NG-boson diagrams tend to be subdominant unless m_{A^0}

is small. It is noticed in Refs. [8, 9] that there are strong cancellation between Barr–Zee diagrams of H^0 involving W^- and G^- loops [10]. However, other contributions dominate over them, and hence this cancellation effect does not appear in the branching ratio.

For comparison, we show the contribution from the one-loop diagrams (Fig. 1) in Fig. 3. It is approximated as

$$A_{\text{one-loop}}^R = \frac{G_F}{8\sqrt{2}\pi^2} \Delta_{\mu e}^L \left[-\frac{\sin \alpha \cos(\alpha - \beta)}{\cos^3 \beta} \frac{m_\mu^2}{m_{h^0}^2} \left(\frac{4}{3} - \log \frac{m_{h^0}^2}{m_\mu^2} \right) + \frac{\cos \alpha \sin(\alpha - \beta)}{\cos^3 \beta} \frac{m_\mu^2}{m_{H^0}^2} \left(\frac{4}{3} - \log \frac{m_{H^0}^2}{m_\mu^2} \right) + \frac{\sin \beta}{\cos^3 \beta} \frac{m_\mu^2}{m_{A^0}^2} \left(\frac{5}{3} - \log \frac{m_{A^0}^2}{m_\mu^2} \right) \right]. \quad (15)$$

As expected, this is always subdominant.

Now we consider competition between the gaugino- and Higgs-mediated contributions to $\mu \rightarrow e\gamma$. The gaugino-mediated contribution to A^R is approximated as

$$A_{\text{gaugino}}^R = \frac{1}{15} \frac{\alpha_2}{4\pi} \left(1 + \frac{5}{4} \tan^2 \theta_W \right) \frac{1}{M_{\text{SUSY}}^2} \delta_{ji}^{LL} \tan \beta, \quad (16)$$

where we take a common value M_{SUSY} for the SUSY particle masses.

Fig. 4 is a contour plot for square of ratio of the Higgs- and gaugino-mediated contributions to A^R as functions of M_{SUSY}/m_{A^0} and $\tan \beta$. The line on which this ratio is equal to unity is boundary of the two regions where each effect dominates $\mu \rightarrow e\gamma$. In small $\tan \beta$ region, the Higgs-mediated contribution comes from mainly Barr–Zee diagrams including top-quark loop. Both Higgs- and gaugino-mediated contribution to A_R are approximately proportional to $\tan \beta$. However, in large $\tan \beta$ region, the Barr–Zee diagrams with bottom-quark and tau-lepton loops receive larger $\tan \beta$ enhancement, and the Higgs-mediated contribution becomes larger with the same M_{SUSY}/m_{A^0} value. We found that the Higgs- and gaugino-mediated effects are comparable to each other in $\mu \rightarrow e\gamma$ when $M_{\text{SUSY}}/m_{A^0} \simeq 50$.

3 Correlation among LFV processes

Now we discuss other μ - e transition processes, $\mu \rightarrow 3e$ and μ - e conversion in nuclei, when the Higgs-mediated contributions are dominant in the MSSM. These two processes have strong correlation with $\mu \rightarrow e\gamma$ when the gaugino-mediated contributions are dominant, since effective LFV dipole operator determines the processes.

First, we consider $\mu \rightarrow 3e$. This process is generated from three types of effective four-Fermi operators; scalar-, vector-, and dipole operators. Scalar operators are induced by tree-level Higgs boson exchange. On the other hand, vector and dipole ones are generated by virtual-photon mediating processes $\mu \rightarrow e\gamma^*$ at higher order. When only the Higgs bosons contribute to $\mu \rightarrow 3e$, vector operator mainly comes from one-loop diagrams, and dipole one is generated by two-loop Barr–Zee diagrams. Since diagrams for vector and scalar operators need two chirality flips, these operators are suppressed by small Yukawa

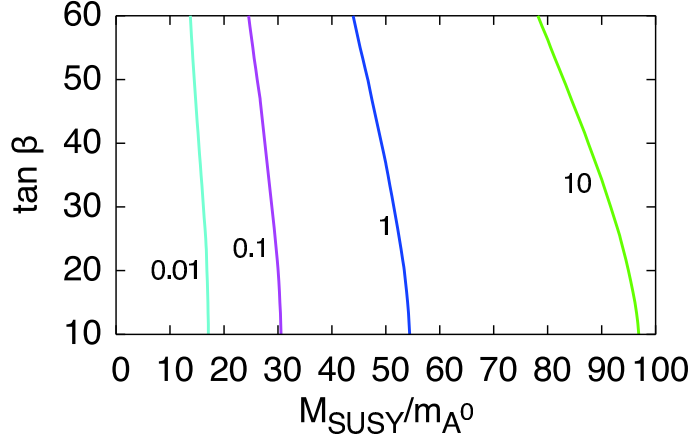


Figure 4: Contour plot of square of ratio for Higgs- and gaugino-mediated contributions to A^R as functions of M_{SUSY}/m_{A^0} and $\tan \beta$. Higgs-mediated contribution is dominant at right-handed side of line on which this ratio is equal to unity.

couplings (y_μ or y_e), compared to dipole operator. Vector and dipole operators come from higher-order effects and they are suppressed by loop factors.

Thus, contributions of these operators are estimated roughly as follows,

$$\begin{aligned}
A_0 &\simeq \frac{y_\mu y_e \Delta_{\mu e}^L}{m_{A^0}^2} \tan^3 \beta, \\
A_1 &\simeq \frac{\alpha_{\text{em}}}{4\pi} \frac{y_\mu^2 \Delta_{\mu e}^L}{m_{A^0}^2} \tan^3 \beta \log \left(\frac{m_\mu^2}{m_{A^0}^2} \right), \\
A_2 &\simeq \left(\frac{\alpha_{\text{em}}}{4\pi} \right)^2 \frac{y_t^2 \Delta_{\mu e}^L}{m_{A^0}^2} \tan \beta \left[\log \left(\frac{m_t^2}{m_{A^0}^2} \right) \right]^2,
\end{aligned} \tag{17}$$

where lower indices (0,1,2) mean coefficients for scalar, vector, and dipole operators, respectively. Ratio of these coefficients becomes $A_0 : A_1 : A_2 \simeq 1 : O(1) : O(10)$, and the coefficient for dipole operator is the dominant contribution. There is also $\log(m_\mu^2/m_e^2)$ enhancement for dipole operator contribution to $\mu \rightarrow 3e$, which comes from final state phase space integral. As a consequence, $\mu \rightarrow 3e$ is dominated by dipole operator and there is strong correlation between $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$,

$$\frac{\text{BR}(\mu \rightarrow 3e)}{\text{BR}(\mu \rightarrow e\gamma)} \simeq \frac{\alpha_{\text{em}}}{3\pi} \left(\log \left(\frac{m_\mu^2}{m_e^2} \right) - \frac{11}{4} \right) \simeq 0.006. \tag{18}$$

Next, we discuss μ - e conversion in nuclei. Dominant contribution to this process comes from tree-level Higgs boson exchange [7]. The reason is that coupling between Higgs boson and nucleon is characterized by the nucleon mass m_N through the conformal anomaly relation [11], and could evade suppression of light constituent quark mass. Then,

branching ratio for μ - e conversion in nuclei at large $\tan\beta$ is derived from formulae in Refs. [12, 13] as follows,

$$\text{BR}(\mu\text{Al} \rightarrow e\text{Al}) \simeq 6.8 \times 10^{-5} \frac{G_F^2 m_\mu^7 m_p^2}{m_{H^0}^4 \omega_{\text{capt}}^{\text{Al}}} (\Delta_{\mu e}^L)^2 \tan^6 \beta. \quad (19)$$

Here, $\omega_{\text{capt}}^{\text{Al}} \simeq 0.7054 \times 10^6 \text{sec}^{-1}$, and m_p is proton mass. We use the recent lattice simulation result [14] for the σ term, which shows that the strange quark content of the nucleon is much smaller than previously thought. Notice that branching ratio for μ - e conversion in nuclei is scaled by $\tan^6 \beta$, while those for $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ are proportional to $\tan^2 \beta$.

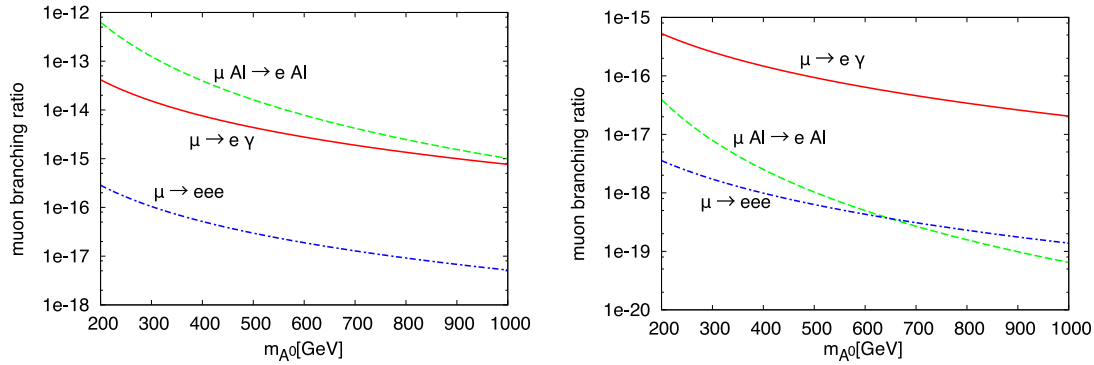


Figure 5: Branching ratios of Higgs-mediated cLFV processes. We took $\tan\beta = 50$ (left) and $\tan\beta = 10$ (right), $\Delta_{\mu e}^L = 5.0 \times 10^{-6}$.

In Fig. 5 branching ratios of Higgs-mediated LFV processes are shown as a function of m_{A^0} . Though we include contributions from the scalar and vector operators in the evaluation of $\text{BR}(\mu \rightarrow 3e)$ in addition to the dipole one, it is found from this figure that there is still tight correlation between $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$, $\text{BR}(\mu \rightarrow e\gamma)/\text{BR}(\mu \rightarrow 3e) \simeq O(\alpha_{\text{em}})$. Thus, it is a signature that dipole operator dominates these two processes. On the other hand, μ - e conversion in nuclei is dominated by tree-level Higgs boson exchange, and such simple correlation does not appear, as expected. In the gaugino-mediation case, the dipole operator dominates three processes. Thus, it is important to measure μ - e conversion rate for discrimination of these two cases, in addition to $\mu \rightarrow e\gamma$.

It is also found that while μ - e conversion process is simply scaled as $1/m_{A^0}^4$, other two processes are not. This is because other two processes receive large logarithmic corrections.

Fig. 6 shows contour plot of $\text{BR}(\mu\text{Al} \rightarrow e\text{Al}) / \text{BR}(\mu \rightarrow e\gamma)$ including both the Higgs- and gaugino-mediated contributions. If the Higgs-mediated contribution is dominant in the cLFV processes, the ratio between $\mu \rightarrow e\gamma$ and $\mu N \rightarrow eN$ is sensitive to $\tan\beta$, but not to M_{SUSY}/m_{A^0} . On the other hand, if gaugino-mediated LFV is dominant, this ratio is about $O(\alpha_{\text{em}})$ since dipole operator contributions dominate the cLFV processes. When

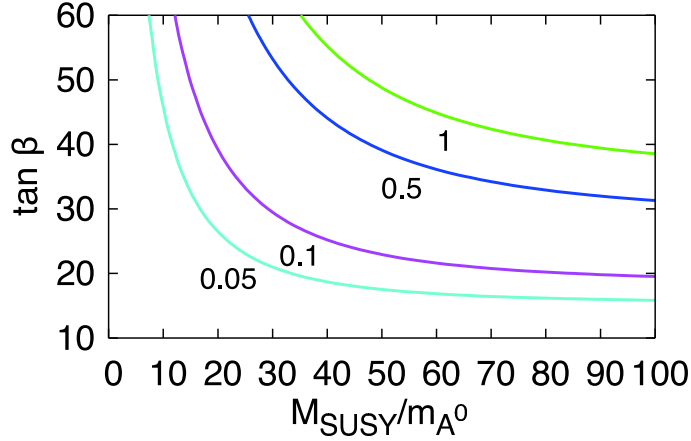


Figure 6: Contour plot of $\text{BR}(\mu\text{Al} \rightarrow e\text{Al}) / \text{BR}(\mu \rightarrow e\gamma)$, $\tan\beta$ vs M_{SUSY}/m_{A^0} including both the Higgs- and gaugino-mediated contributions.

$M_{\text{SUSY}}/m_{A^0} \sim (10 - 50)$ and $\tan\beta \gtrsim 10$, both Higgs- and gaugino-mediated diagrams contribute to those processes in different way and we could give constraints M_{SUSY}/m_{A^0} and $\tan\beta$ from $\text{BR}(\mu\text{Al} \rightarrow e\text{Al})/\text{BR}(\mu \rightarrow e\gamma)$.

4 Conclusions and discussion

In this letter, we reevaluated μ - e transition processes induced by non-holomorphic Yukawa interactions in the MSSM. We discussed correlation among branching ratios for $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, and μ - e conversion in nuclei in the MSSM, by including both the gaugino- and Higgs-mediated contributions to the processes. It was assumed in this letter that left-handed sleptons have flavor-mixing mass terms.

Though Higgs-mediated contribution to μ - e transition processes is evaluated in previous works [8, 9], we found that contribution from Barr-Zee diagram including W^- boson, which was thought to be the largest contribution to $\mu \rightarrow e\gamma$ among various Higgs-mediated contributions, has incorrect dependence on $\tan\beta$. As a result, branching ratio for $\mu \rightarrow e\gamma$ was overestimated. We showed that Barr-Zee diagrams including top quark are rather dominant, and those including bottom quark and tau lepton are also sizable only when $\tan\beta$ is large. Then, the Higgs-mediated contribution dominates over the gaugino-mediated one in $\mu \rightarrow e\gamma$ when $M_{\text{SUSY}}/m_{A^0} \gtrsim 50$, irrespectively of $\tan\beta$ as far as $\tan\beta$ is not large.

We evaluated ratio of branching ratios for $\mu \rightarrow e\gamma$ and μ - e conversion in nuclei by including both the gaugino- and Higgs-mediated contributions to the processes. We found that the ratio is sensitive to $\tan\beta$ and M_{SUSY}/m_{A^0} when $M_{\text{SUSY}}/m_{A^0} \sim (10 - 50)$ and $\tan\beta \gtrsim 10$. Ratio of the branching ratios for $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ is insensitive to $\tan\beta$ and the MSSM mass spectrum, since the dipole term contribution is always dominant in $\mu \rightarrow 3e$.

In general, right-handed slepton mass terms or A terms could be sources for flavor-mixing. In particular, gaugino-mediated contributions from right-handed slepton mass receive destructive interference between the bino and bino-Higgsino amplitudes [15]. Therefore, Higgs-mediated contribution may be significant in some parameter region and decoupling behavior of M_{SUSY} could be modified. We leave it for our future work.

A Barr–Zee integrals

The Barr–Zee integrals $f(z)$, $g(z)$ and $h(z)$ are given by

$$\begin{aligned} f(z) &= \frac{1}{2}z \int_0^1 dx \frac{1-2x(1-x)}{x(1-x)-z} \log \frac{x(1-x)}{z} , \\ g(z) &= \frac{1}{2}z \int_0^1 dx \frac{1}{x(1-x)-z} \log \frac{x(1-x)}{z} , \\ h(z) &\left(= z^2 \frac{d}{dz} \left(\frac{g(z)}{z} \right) \right) \\ &= \frac{z}{2} \int_0^1 \frac{dx}{z-x(1-x)} \left[1 + \frac{z}{z-x(1-x)} \log \frac{x(1-x)}{z} \right] . \end{aligned} \tag{20}$$

In the limit of $1 \gg z$, the asymptotic forms of them is given as follows [10],

$$\begin{aligned} f(z) &\sim \frac{z}{2}(\log z)^2 , \quad g(z) \sim \frac{z}{2}(\log z)^2 , \quad h(z) \sim z \log z , \\ f(z) - g(z) &\sim z(\log z + 2) . \end{aligned} \tag{21}$$

On the other hand, in the limit of $1 \ll z$,

$$\begin{aligned} f(z) &\sim \frac{1}{3} \log z + \frac{13}{18} , \quad g(z) \sim \frac{1}{2} \log z + 1 , \quad h(z) \sim -\frac{1}{2}(\log z + 1) , \\ f(z) - g(z) &\sim -\frac{1}{6} \log z - \frac{5}{18} . \end{aligned} \tag{22}$$

Similarly, in the limit of $1 \gg z$, the asymptotic forms of $F(z) = 3f(z) + 5g(z) + 3/4g(z) + 3/4h(z)$ and $F'(z) = (g(z) - f(z))/(2z)$ are derived as

$$\begin{aligned} F(z) &\sim \frac{35}{8}z(\log z)^2 + \frac{3}{4}z \log z , \\ F'(z) &\sim -\frac{1}{2}(\log z + 2) . \end{aligned} \tag{23}$$

On the other hand, in the limit of $1 \ll z$,

$$\begin{aligned} F(z) &\sim \frac{7}{2} \log z + \frac{181}{24} , \\ F'(z) &\sim \frac{1}{2z} \left(\frac{1}{6} \log z + \frac{5}{18} \right) . \end{aligned} \tag{24}$$

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References

- [1] For a recent review, M. Raidal *et al.*, Eur. Phys. J. C **57** (2008) 13 [arXiv:0801.1826 [hep-ph]].
- [2] J. Adam *et al.* [MEG collaboration], Nucl. Phys. B **834** (2010) 1 [arXiv:0908.2594 [hep-ex]].
- [3] R. Bernstein, talk given in the 4th International Workshop on Nuclear and Particle Physics at J-PARC (NP08), Mito, Ibaraki, Japan, March, 2008 (<http://j-parc.jp/NP08/>).
- [4] A. Sato, talk given in the 4th International Workshop on Nuclear and Particle Physics at J-PARC (NP08), Mito, Ibaraki, Japan, March, 2008 (<http://j-parc.jp/NP08/>).
- [5] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57** (1986) 961; J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B **357** (1995) 579 [arXiv:hep-ph/9501407]; J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D **53** (1996) 2442 [arXiv:hep-ph/9510309]; J. Hisano and D. Nomura, Phys. Rev. D **59** (1999) 116005 [arXiv:hep-ph/9810479]; J. A. Casas and A. Ibarra, Nucl. Phys. B **618** (2001) 171 [arXiv:hep-ph/0103065].
- [6] K. S. Babu and C. Kolda, Phys. Rev. Lett. **89** (2002) 241802 [hep-ph/0206310].
- [7] R. Kitano, M. Koike, S. Komine and Y. Okada, Phys. Lett. B **575** (2003) 300 [arXiv:hep-ph/0308021].
- [8] P. Paradisi, JHEP **0608** (2006) 047 [arXiv:hep-ph/0601100].
- [9] P. Paradisi, JHEP **0602** (2006) 050 [arXiv:hep-ph/0508054].
- [10] D. Chang, W. S. Hou and W. Y. Keung, Phys. Rev. D **48** (1993) 217 [arXiv:hep-ph/9302267].
- [11] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. B **78** (1978). 443

- [12] R. Kitano, M. Koike and Y. Okada, Phys. Rev. D **66** (2002) 096002 [Erratum-ibid. D **76** (2007) 059902] [arXiv:hep-ph/0203110].
- [13] V. Cirigliano, R. Kitano, Y. Okada and P. Tuzon, Phys. Rev. D **80** (2009) 013002 [arXiv:0904.0957 [hep-ph]].
- [14] H. Ohki *et al.*, Phys. Rev. D **78** (2008) 054502 [arXiv:0806.4744 [hep-lat]].
- [15] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Lett. B **391** (1997) 341 [Erratum-ibid. B **397** (1997) 357] [arXiv:hep-ph/9605296].